Problem #4

Assume that the economy in a given country consists of only two persons. It is only possible to produce two goods (drill drivers and markers). Production possibilities frontiers of these persons are given by: $D + M = 40$ and $D + 2M = 60$. Because of close relationships between the persons composing this economy, we can only speak of utility from the point of view of their joint consumption.

a) Provide the production possibilities frontier in algebraic and graphical form.

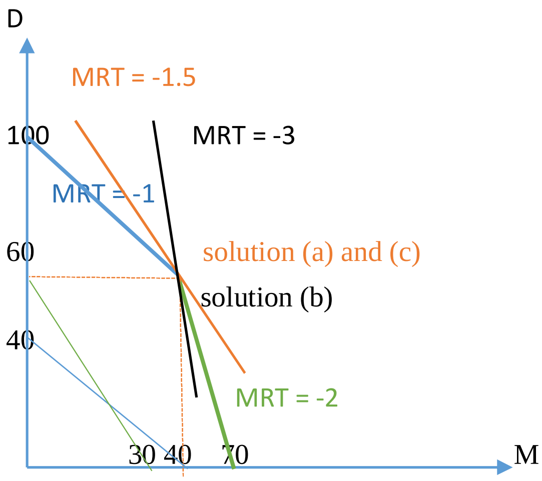
b) Assuming that the utility function is given by the formula $U(D,M) = DM^2$, find the optimum consumption level of both goods.

c) Assuming that the utility function is given by the formula $U(D,M) = DM$, find the optimum consumption level of both goods.

Solution

a) Person 1: D+M = 40 ⇒ D=40-M (slope = -1) ⇒ $MRT_{MD}=MC_M/MC_D=-1$ ⇒ This person can produce max 40 units of D or max 40 units of M

Person 2: D+2M = 60 ⇒ D=60-2M (slope = -2) ⇒ $MRT_{MD}=MC_M/MC_D=-2$ ⇒ $MC_M>MC_D$ ⇒ D=2M ⇒ comparative advantage in D



The production possibility frontier:

D = 100 – M for 0≤M<40

D = 140 – 2M for 40<M≤ 70

(D+M)+(D+2M) = 40+60

2D+3M = 100

$D = 50 - \frac{3}{2}M$ for M=40 ⇒ $MRT_{MD}=MC_M/MC_D = -\frac{3}{2}$

Conclusion: Person 1 will produce M, i.e. $\omega_{M1} = 40$ and $\omega_{D1} = 0$. Person 2 will produce D, i.e. $\omega_{M2} = 0$ and $\omega_{D2} = 60$. The exchange rate is $\frac{P_M}{P_D}$=1.5

b) $U(D, M) = DM^2$

$MRS_{MD} = -\frac{2DM}{M^2} = -\frac{2D}{M}$

Since endowment is already set in (a), we can use now a pure exchange model to determine consumption allocation.

$MRT_{MD} = \frac{P_M}{P_D} = \frac{3}{2} = \frac{2D}{M} = MRS_{MD}$ ⇒ $D = \frac{3}{4}M$

⇓

Budget constraint for Person 1: $P_M*M_1 + P_D*D_1 = P_M*40$ Budget constraint for Person 2: $P_M*M_2 + P_D*D_2 = P_D*60$

$M_1 + \frac{P_D}{P_M}$*D_1 = 40

$M_1 + \frac{2}{3}$*D_1 = 40

$M_1 + \frac{2}{3}$*$\frac{3}{4}$ *M_1 = 40

1.5*M_1 = 40

M_1 = 26$\frac{2}{3}$ $\Rightarrow D_1 = \frac{3}{4} M_1 = 20$

$\frac{P_M}{P_D}$*M_2 + D_2 = 60

$\frac{3}{2}$ *M_2 + D_2 = 60

$\frac{3}{2}$ *$\frac{4}{3}$ *D_2 + D_2 = 60

3*D_2 = 60

D_2 = 20 $\Rightarrow M_2 = \frac{4}{3} D_2 = 26\frac{2}{3}$

Check the Walras Law: $z_M(M_1, M_2) = 0$ and $z_D(D_1, D_2) = 0$

$z_M(M_1, M_2) = M_1 + M_2 - \omega_{M1} - \omega_{M2} = 26\frac{2}{3} + 26\frac{2}{3} - 40 - 0 > 0 \Rightarrow$ excess demand on M

$z_D(D_1, D_2) = D_1 + D_2 - \omega_{D1} - \omega_{D2} = 20 + 20 - 0 - 60 < 0 \Rightarrow$ excess supply on D

This means that both consumers strongly prefer M, but the exchange rate is too low to find the equilibrium.

Person 1 will sell only (40-26$\frac{2}{3}$) units of M, but Person 2 wishes to buy this good according to $MRS_{MD} = \frac{2D}{M} = \frac{P_M}{P_D}$ $\Rightarrow$
$M_2 = 2*D_2*\frac{P_D}{P_M} = 40*\frac{P_D}{P_M}$ $\Rightarrow$ $z_M(M_1, M_2) = 26\frac{2}{3} + 40*\frac{P_D}{P_M} - 40 - 0 = 0$

$\frac{P_D}{P_M} = \frac{1}{3}$ $\Rightarrow$ **$M_2 = 13\frac{1}{3}$**

Person 2 will sell (60-20) units of D, because he just wishes to consume 20 units of this product. Person 1 wishes to buy this good according to $MRS_{MD} = \frac{2D}{M} = \frac{P_M}{P_D}$ $\Rightarrow$ $D_1 = M_1*\frac{P_M}{2P_D} = 40$

Conclusion: Person 1 will consume $(26\frac{2}{3}, 40)$ and Person 2 $(13\frac{1}{3}, 20)$. The exchange rate will be changed to $\frac{P_M}{P_D}$=3, because both persons have the same preferences with a strong priority to M.

c) U(D, M) = DM

$MRS_{MD} = -\frac{D}{M}$

Since endowment is already set in (a), we can use now a pure exchange model to determine consumption allocation.

$MRT_{MD} = \frac{P_M}{P_D} = \frac{3}{2} = \frac{D}{M} = MRS_{MD}$ $\Rightarrow$ $D = \frac{3}{2}M$

Budget constraint for Person 1: $P_M*M_1 + P_D*D_1 = P_M*40$

$M_1 + \frac{P_D}{P_M}$*D_1 = 40

$M_1 + \frac{2}{3}$*D_1 = 40

$M_1 + \frac{2}{3}$*$\frac{3}{2}$ *M_1 = 40

2*M_1 = 40

$M_1 = 20 \Rightarrow D_1 = \frac{3}{2} M_1 = 30$

Budget constraint for Person 2: $P_M*M_2 + P_D*D_2 = P_D*60$

$\frac{P_M}{P_D}$*M_2 + D_2 = 60

$\frac{3}{2}$ *M_2 + D_2 = 60

$\frac{3}{2}$ *$\frac{2}{3}$ *D_2 + D_2 = 60

2*D_2 = 60

$D_2 = 30 \Rightarrow M_2 = \frac{2}{3} D_2 = 20$

Check the Walras Law: $z_M(M_1, M_2) = 0$ and $z_D(D_1, D_2) = 0$

$z_M(M_1, M_2) = M_1 + M_2 - \omega_{M1} - \omega_{M2} = 20 + 20 - 40 - 0 = 0$

$z_D(D_1, D_2) = D_1 + D_2 - \omega_{D1} - \omega_{D2} = 30 + 30 - 0 - 60 = 0$

Conclusion: Consumers will have the symmetric allocation $(20, 30)$, because both persons have the same preferences with no strong priority to neither of goods. Thus the exchange rate will be the same as in (a).